Trends and Stationarity in Global Temperature Data

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Abstract

Interpretation of the effects of increasing atmospheric carbon dioxide on temperature is made more difficult by the fact that it is unclear whether sufficient global warming has taken place to allow a statistically significant finding of any upward trend in the temperature series. We add to the few existing statistical results by reporting tests for both deterministic and stochastic non-stationarity (trends) in time series of global average temperature. We conclude that the statistical evidence is sufficient to reject the hypothesis of a stochastic trend; however, there seems to be more evidence of a trend which could be approximated by a deterministic linear model.

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Introduction

Among the methods that have been used to investigate a possible link between atmospheric carbon dioxide concentration and global average temperatures, there have been a number of statistical studies aimed at finding a statistically significant relationship in long time series of the two variables. Because one hypothesis of interest (that of a global trend toward warming, whether or not related to atmospheric CO_2) implies non-stationary temperature data, it is important that careful statistical work take account of the voluminous recent literature on the treatment of non-stationary time series data; see in particular Fuller (1976), Dickey and Fuller (1979), Said and Dickey (1984), Phillips (1986, 1987).

An examination of atmospheric CO_2 data clearly suggests a fairly regular seasonal cycle around a marked increasing trend. In the case of global temperature data, however, the underlying pattern is much less clear. Moreover, the nature of the stochastic process generating temperature data is crucial to the statistical examination of a possible link between that variable and atmospheric CO_2. If fluctuations in temperature are simply the random fluctuations of a stationary time series, then there is no global warming *trend* to be explained, by CO_2 concentrations or by any other cause. If there is statistical evidence of an increasing trend in global temperature, however, then there are a number of methods by which to investigate a possible relationship between two non-stationary series which may be applied. There are also well-known pitfalls in attempting to identify such relationships (beginning with Yule (1926)). Phillips (1986) points out the inadequacy of deterministic de-trending methods if the non-stationarity springs from a stochastic trend[1]; for series which do contain stochastic trends, a long-term relationship can be modelled as a *co-integrating* relationship (Engle and Granger (1987)) or *common trend* (Stock and Watson (1988)), implying a linear combination of the series which has a lower order of integration than do the individual data series.

This question of the stationarity or non-stationarity of the global average temperature series has been addressed by, *inter alia*, Solow (1987) and Solow and Broadus (1989). The first of these studies uses the two-phase regression model to test for a possible break in trend; Solow is unable to reject the hypothesis of no change in the temperature series. Solow and Broadus reach a similar conclusion based on a less formal examination of the temperature data. Kuo, Lindberg and Thomson (1990) find evidence of coherence between the spectra of the CO_2 concentration and global average temperature series. However, the latter authors do not consider the possibility of a stochastic, rather than deterministic trend in the underlying series; the deterministic linear de-trending which they apply can produce misleading results if the underlying trend is stochastic (again see, *e.g.*, Phillips (1986)).

The present paper investigates the question of the stationarity

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1 By "stochastic trend" we mean a process containing a latent root in its autoregressive polynomial which lies outside the unit circle. A series with a "deterministic trend" is one which could be approximated as the sum of a stationary series and a deterministic part which is tending to increase (or decrease) over time. Both are non-stationary processes.

or non-stationarity of global temperature data by applying tests for *stochastic* non-stationarity (in the form of a latent root of the autoregressive polynomial which lies on the unit circle), and for a particular deterministic non-stationarity. The statistical tests for the former are those of Dickey and Fuller (1979), a t-type test using a Monte Carlo tabulation of the appropriate non-standard distribution.

The results of this study may be viewed as complementary to those of Solow (1987), who tests a related deterministic-trend hypothesis. Solow tests for a change in a deterministic trend, and finds no evidence of such change. These results leave unanswered the question of whether the (presumably unbroken) trend is positive to a statistically significant degree, or whether the series is stationary. We attempt to answer this question in section 3, with the results of section 2 forming a necessary precursor.

2. Data and testing for a stochastic trend

The data used here were kindly provided by James Hansen and Jeffrey Jonas of the Goddard Space Flight Center, Institute for Space Studies, and are described in Hansen and Lebedeff (1987). The series of global temperature "changes"[2] runs in monthly increments from 1880 to 1988, yielding a total of 1308 observations. The data are also available for particular regions, and we report statistics for the Northern and Southern hemisphere averages as well; Figures 1 to 3

2 Hansen and Lebedeff use the symbol ΔT, but the series should *not* be confused with one of first differences, $T_\tau - T_{\tau-1}$. Each entry represents an average temperature in the region, scaled such that the recorded value is the deviation in Celcius degreesfrom the average value for that month over a baseline period. Hence the series is a level series, non-stationary under the null hypothesis. For further information, see Hansen and Lebedeff (1987).

represent these data graphically.

The test for stochastic non-stationarity that we apply is described in Fuller (1976) and Dickey and Fuller (1979). The null hypothesis is of a root of unity in the autoregressive representation of the time series; in AR(1) form[3],

$$Y_t = Y_{t-1} + u_t, \quad \alpha(L)u_t = \beta(L)\varepsilon_t, \tag{1}$$

where ε_t is a white-noise process and the latent roots of the autoregressive polynomial $\alpha(L)$ lie within the unit circle. The test statistic is the conventional "t"-statistic on γ_0 in the regression

$$\Delta Y_t = \alpha + \gamma_0 Y_{t-1} + \sum_{i=1}^{k} \gamma_i \Delta Y_{t-i} + \nu_t. \tag{2}$$

Under the null hypothesis, $\gamma_0 = 0$. However the distribution of the statistic is *not* the standard t-distribution; Fuller (1976) tabulates the percentiles of the distribution for the model (2) and two variants. The number of lagged differences, k, is chosen to render $\{\nu_t\}_1^T$ a white noise process; Said and Dickey (1984) provide bounds on the appropriate rate of increase of k with the sample size when these lagged values are used to capture moving-average, as well as autoregressive, components in the underlying error process $\{u_t\}_1^T$.

Table 1 reports test statistics and critical values for the model (2) applied to the global average temperature series. The addition of a linear (deterministic) time trend to the model (2) requires a change in the critical values applied, which is reflected in the values reported in Table 1 for the modified model:

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3 Test statistics are reported for the data in level form. The logarithmic transformation makes only minimal changes in test-statistics (typically in the second decimal place).

$$\Delta Y_t = \alpha + \beta t + \gamma_0 Y_{t-1} + \sum_{i=1}^{k} \gamma_i \Delta Y_{t-i} + \nu_t, \qquad (2')$$

t = 1,2,...T, where T = 1308 is the sample size. Lagrange multiplier tests for autocorrelation (not reported) indicate that k=4 is sufficient in most cases to produce a residual error term not significantly different from white noise.

Table 1
Augmented Dickey-Fuller Statistics
H_0: $\gamma_0 = 0$.

Model: (2) k	Percentile of distribution[4] 5%	2.5%	1%	test statistic North	South	Global
2	-2.86	-3.12	-3.43	-9.78	-10.60	-8.27
4	-2.86	-3.12	-3.43	-8.09	-8.28	-6.78
8	-2.86	-3.12	-3.43	-5.12	-5.50	-4.45

Model: (2') k	Percentile of distribution 5%	2.5%	1%	test statistic North	South	Global
2	-3.41	-3.66	-3.96	-12.90	-13.70	-11.68
4	-3.41	-3.66	-3.96	-11.23	-11.24	-10.10
8	-3.41	-3.66	-3.96	-7.70	-8.10	- 7.18

Values of $\hat{\gamma}_0$

k:	Model: (2) North	South	Global	Model: (2') North	South	Global
2	-0.22	-0.28	-0.16	-0.36	-0.44	-0.31
4	-0.19	-0.23	-0.14	-0.35	-0.41	-0.29
8	-0.13	-0.17	-0.10	-0.28	-0.34	-0.23

4 See Fuller (1976) p. 371 ff. for the full set of percentiles of this distribution under the null hypothesis.

As Table 1 indicates, the null hypothesis of a unit root in the autoregressive polynomial is given a very low probability by the test,and the deviation from the null is in the direction of stationarity. On the existing sample, then, average temperature data are consistent with a strongly autoregressive underlying process, but one which does not accumulate the effects of past stochastic shocks indefinitely. This fact is important in interpreting the results of the next section.

3. Testing for a linear deterministic trend

A test for the presence of a linear trend must also account for the fact that the distribution of the "t"-statistic on a trend term in a linear regression has a non-standard distribution[5] when a stochastic trend is present; even in the absence of a stochastic trend the statistic has a non-standard distribution *in finite samples* if the process is strongly autoregressive. However, the results of section 2 tell us that an underlying stochastic trend is unlikely. Moreover, adding Y_{t-1} to both sides of models (2) and (2') to transform to AR form, the first autoregressive parameter is given by $1+\gamma_0$ and we see that values of $1+\gamma_0$ range from 0.56 (Model (2'), k = 2, Southern hemisphere) to 0.90 (Model (2), k = 8, Global average). On a sample of the size available here, this parameter is sufficiently far from unity that we can easily rely on the standard (in this case

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5 The distribution of the statistic on a linear trend term is in fact non-degenerate for a stochastically trending variable which contains no linear trend in the true process: Phillips (1986).

asymptotic) t-distribution for the t-statistic on the trend term in the model, ignoring finite-sample distortions[6].

Solow (1987) treated the temperature series as possibly containing a purely deterministic linear trend, and asked whether any break in trend could be detected. While no such break could be found, the possibility remains that an unbroken trend in temperature affected the entire data series. In this case we would again say that stationarity fails, and the possibility that such a break could be attributed to increasing atmospheric CO_2 concentration remains. To test this possibility, we return to the model (2′), or to the transformed version

$$Y_t = \alpha + \beta t + (1+\gamma_0)Y_{t-1} + \sum_{i=1}^{k}\gamma_i \Delta Y_{t-i} + \nu_t. \tag{3}$$

Table 2 contains the estimates of β on the various samples and the corresponding t-statistics.

Table 2
Tests of Significance of Linear Trend
H_0: $\beta = 0$

Model: (3) k	North $\hat{\beta}$	North $t_{(\beta=0)}$	South $\hat{\beta}$	South $t_{(\beta=0)}$	Global $\hat{\beta}$	Global $t_{(\beta=0)}$
2	0.0188	8.12	0.0153	8.33	0.0143	8.04
4	0.0185	7.61	0.0142	7.41	0.0138	7.37
8	0.0149	5.70	0.0120	5.88	0.0111	5.61

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6 The importance of the small-sample distortion to the distribution is easily established by small Monte Carlo or bootstrap experiments. The effect gains in importance as the largest root of the AR polynomial approaches one, and diminishes as the sample size grows.

Table 2 indicates that there is sufficient evidence on this sample to reject a null hypothesis of no (linear) deterministic trend at conventional significance levels. A natural question to pose in interpreting this result is that of whether or not this trend rate of increase was approximately constant over the sample, an effect which one might attempt to detect through a test for a change in a linear trend at an unknown point in the sample. Solow (1987) provides just such a test statistic which, again, does not offer strong evidence of a change in trend.

4. Conclusion

The statistical literature on the detection of trends in time series makes an important distinction between a stochastic trend, with which the tendency for a series to wander from any particular unconditional distribution function depends upon realisations of a random variable, and a deterministic trend in which the evolution of the unconditional distribution function over time is predictable. The application of tests for such trends requires, in many instances, the use of non-standard distributions functions for test statistics.

In the case of global temperature data, we do not find evidence of a stochastic trend in the form of a process with a root exceeding unity in an autoregressive lag polynomial. This result allows us to apply a test for a linear deterministic trend which would otherwise be invalid, and in so doing to extend the earlier results of Solow (1987). We do find some evidence of a small deterministic trend which can be approximated by a linear term.

We note that, while these results in themselves imply nothing about the link between CO_2 concentration and temperature, there is evidence of gradually increasing concentration of atmospheric CO_2 over a long period beginning in the 19^{th} century. It follows that an effect of greenhouse gases on temperature, if it were present, need not necessarily show up in the form of a *change* in trend over the sample examined here, but could instead imply a uniform trend over the entire period. A statistical examination of this link, however, requires a further careful study of long-term movements in the two series of interest, an exercise made more difficult by the relatively short span of measurements of atmospheric CO_2 concentration.

Figure 1: Global Temperature

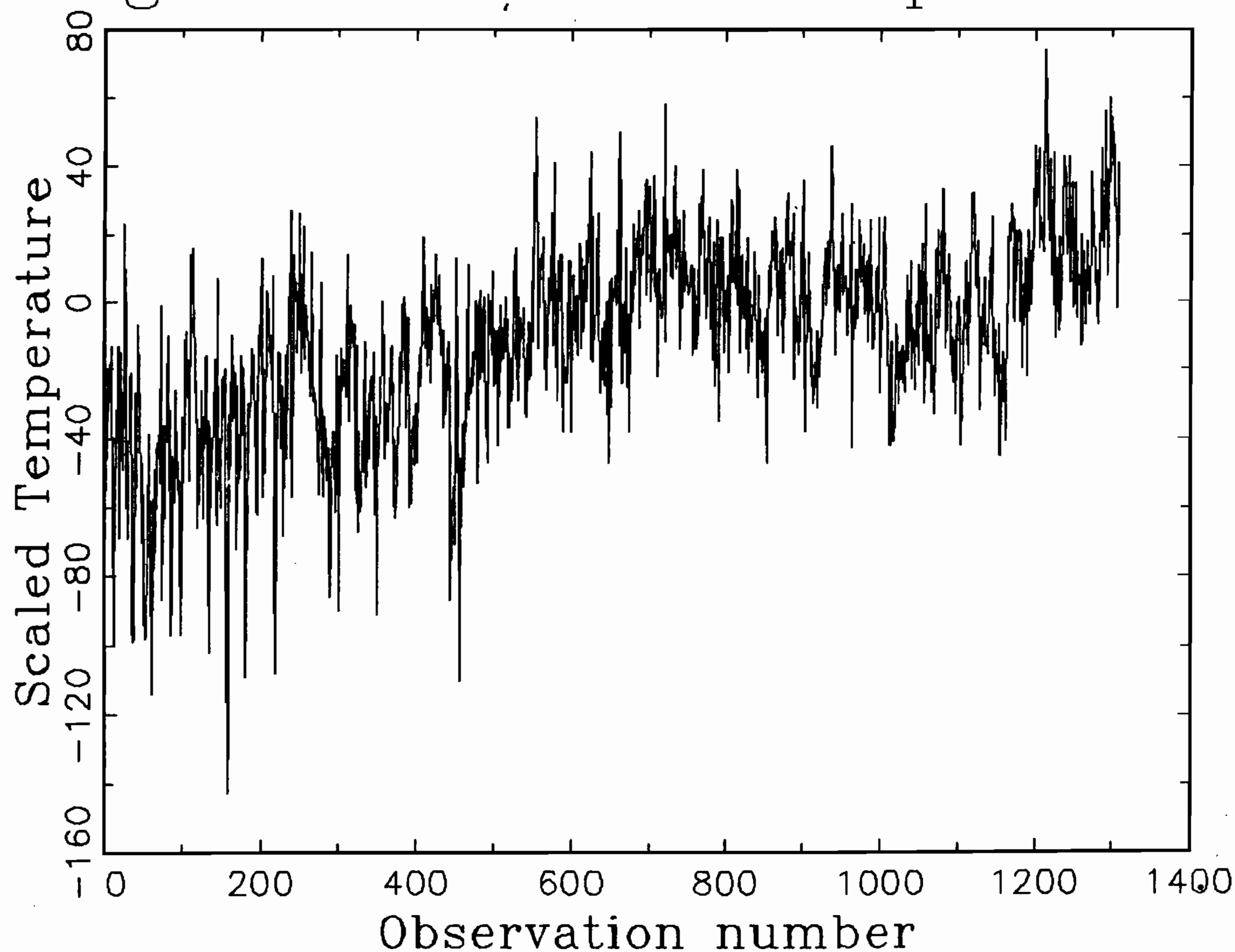

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Figure 2: Northern

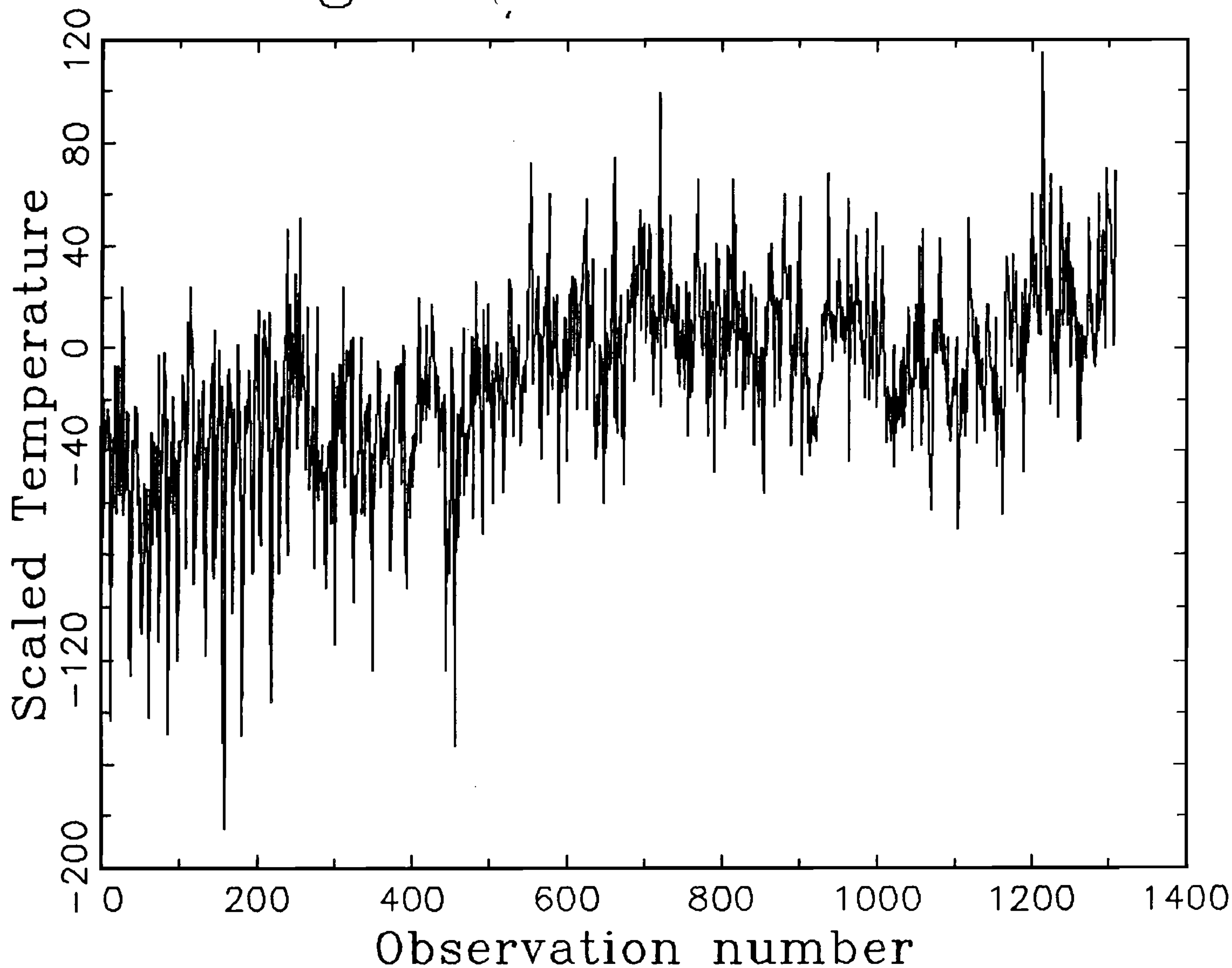

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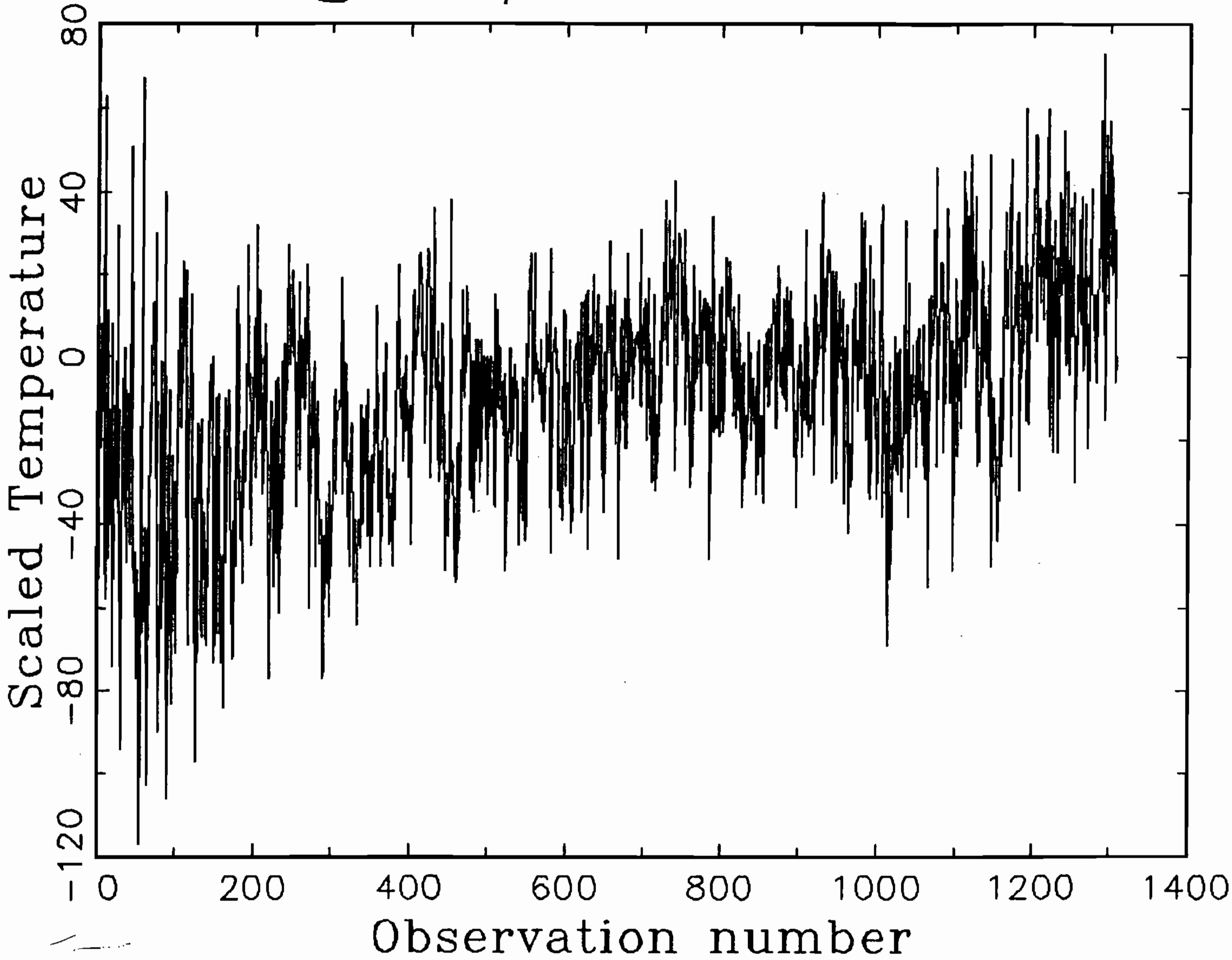
Figure 3: Southern



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